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The Electroweak Interactions as a Confinement Phenomenon.

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Abstract

We consider a model for the electroweak interactions based on the assumption that physical particles are singlets under the gauge group $SU(2)$. The concept of complementarity explains why the standard model works with such an extraordinary precision although the fermions and bosons of the model can be viewed as composite objects of some more fundamental fermions and bosons. We study the incorporation of QED in the model. Furthermore we consider possible deviations from the standard model at very high energies, e.g. excited states of the weak bosons.

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1 Introduction

The standard model of the basic interactions consists of two sectors, the QCD-sector based on an unbroken and confining gauge theory in color space, and the electroweak sector [1], based on the gauge group $SU(2)_L \otimes U(1)_Y$, which is spontaneously broken. In QCD the gauge bosons (gluons) are massless but three of the gauge bosons of the electroweak sector acquire masses through the spontaneous symmetry breaking. This looks like a peculiar asymmetry between the strong and electroweak sectors, and many authors have attempted to avoid it by extending the standard model, e.g. by considering composite models [2] for leptons, quarks and the weak bosons.

As in particular emphasized by 't Hooft, the asymmetry between the two sectors is, in fact, much less pronounced if one views the electroweak gauge model from a different point of view [3], using the idea of complementarity [4, 5]. Like in QCD, the electroweak sector can be built upon a confining gauge theory. But in contrast to QCD such a theory does not show a clear-cut distinction between the confining phase and the Higgs phase, like there is no such distinction between the gaseous and liquid phases of water. A continuous transition between the two phases is possible.

In this paper we take this change of viewpoint seriously and suggest that the observed electroweak interactions are indeed due to a confining gauge theory. As long as the confinement scale is disregarded, the theory is identical to the standard model of the electroweak interactions as emphasized in ref. [3]. But we shall demonstrate that the observations, in particular the structure of the neutral current interaction and the strength of the electroweak mixing angle θ_W, suggest that new non-perturbative effects (e.g. bound state effects) might show up at the energy scale of the order of several hundred GeV.

2 The electroweak gauge group and confinement

We start by writing down the Lagrangian of the electroweak standard model, taking into account only the first family of leptons and quarks:

$$\begin{aligned}\mathcal{L}_h = & -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} + \bar{L}_L i\not{D} L_L + \bar{Q}_L i\not{D} Q_L + \bar{e}_R i\not{D} e_R \\ & + \bar{u}_R i\not{D} u_R + \bar{d}_R i\not{D} d_R + G_e \bar{e}_R(\tilde{\phi} L_L) + G_d \bar{d}_R(\tilde{\phi} Q_L)\end{aligned} \qquad (1)$$

$$+G_u\bar{u}_R(\phi Q_L)+h.c.+\frac{1}{2}(D_\mu\phi)^\dagger(D^\mu\phi)-\frac{\mu^2}{2}\phi\phi^\dagger-\frac{\lambda}{4}(\phi\phi^\dagger)^2.$$

The covariant derivative is given by:

$$D_\mu=\partial_\mu-i\frac{g'}{2}Y\mathcal{A}_\mu-i\frac{g}{2}\tau^a B_\mu^a. \tag{2}$$

The field strength tensors are as usual

$$F_{\mu\nu}^a=\partial_\mu B_\nu^a-\partial_\nu B_\mu^a+g\epsilon^{abc}B_\mu^b B_\nu^c \tag{3}$$
$$f_{\mu\nu}=\partial_\mu\mathcal{A}_\nu-\partial_\nu\mathcal{A}_\mu. \tag{4}$$

We have used the definitions:

$$L_L=\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}, Q_L=\begin{pmatrix}u_L\\ d_L\end{pmatrix}, \phi=\begin{pmatrix}\phi^0\\ \phi^-\end{pmatrix} \text{ and } \tilde{\phi}=i\sigma_2\phi^*=\begin{pmatrix}\phi^+\\ -\phi^{0*}\end{pmatrix}. \tag{5}$$

According to the well-known mechanism of spontaneous symmetry breaking the neutral component of the scalar field ϕ^0 acquires a non-vanishing vacuum expectation value v, which gives rise in particular to the masses of the electroweak gauge bosons.

The Lagrangian (1) admits a much wider class of solutions, if we vary the parameters μ^2 and λ arbitrarily. If μ^2 is positive, the $SU(2)_L$ gauge bosons remain massless and lead to a confinement of the corresponding electroweak charges. As pointed out by Osterwalder and Seiler [4] and, using the lattice approach, by Fradkin and Shenker [5], there is no phase transition between the Higgs phase and the confinement phase if the theory has a scalar field in the fundamental representation of the gauge group. However, they were restricting themselves to the case of an $SU(2)$ gauge group without fermions and using the so-called frozen Higgs approximation.

The absence of a phase transition allows one to carry out a smooth transition between the Higgs phase and the confinement phase, a phenomenon denoted as complementarity. However some care has to be taken with the notion of complementarity since it was shown by Damgaard and Heller [6], performing a mean field analysis, that for certain values of the parameters a phase transition can appear. We assume that the parameters in reality are such that the concept of complementarity can be applied.

Using this phenomenon we can start out from the electroweak Lagrangian (1) and study the confinement phase. In particular we shall concentrate on

possible departures from the electroweak standard model at high energies and on the incorporation of the $U(1)_Y$ hypercharge which leads to the electromagnetic interaction.

In the confinement phase the physical particles are $SU(2)_L$ singlets. We introduce the following fundamental left-handed dual-quark doublets, which we denote as D-quarks:

leptonic D-quarks $\quad l_i = \begin{pmatrix} l_1 \\ l_2 \end{pmatrix}$ $\quad$ (spin 1/2, left-handed)

hadronic D-quarks $\quad q_i = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$ $\quad$ (spin 1/2, left-handed, $SU(3)_c$ triplet)

scalar D-quarks $\quad h_i = \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$ $\quad$ (spin 0).

The right-handed particles are those of the standard model. We can identify the left-handed fermions and electroweak bosons of the standard model as bound states:

$$\begin{aligned}
\text{neutrino}: &\quad \nu_L \propto \bar{h}l \\
\text{electron}: &\quad e_L \propto hl \\
\text{up type quark}: &\quad u_L \propto \bar{h}q \\
\text{down type quark}: &\quad d_L \propto hq \\
\text{Higgs particle}: &\quad \phi \propto \bar{h}h, \quad s\text{-wave} \\
W^3-\text{boson}: &\quad W^3 \propto \bar{h}h, \quad p\text{-wave} \\
W^--\text{boson}: &\quad W^- \propto hh, \quad p\text{-wave} \\
W^+-\text{boson}: &\quad W^+ \propto (hh)^\dagger, \quad p\text{-wave}.
\end{aligned} \tag{6}$$

These bound states have to be normalized properly. We shall consider this issue later on. Using a non-relativistic notation, we can say that the scalar Higgs particle is a $\bar{h}h$-state in which the two constituents are in an s-wave. The W^3-boson is the orbital excitation (p-wave). The W^+ (W^-)-bosons are p-waves as well, composed of (hh) $(\bar{h}\bar{h})$ respectively. Due to the $SU(2)$ structure of the wave function there are no s-wave states of the type (hh) or $(\bar{h}\bar{h})$.

As usual in a quantum field theory, the problem is to identify the physical

degrees of freedom. To do so we have to choose the gauge in the appropriate way. The Higgs doublet can be used to fix the gauge. Using the gauge freedom of the local $SU(2)$ group we perform a gauge rotation such that the scalar doublet takes the form:

$$h_i = \begin{pmatrix} F + h_{(1)} \\ 0 \end{pmatrix}, \tag{7}$$

where the parameter F is a real number. If F is sufficiently large we can perform an $1/F$ expansion for the fields defined above. We have

$$\begin{aligned}
\nu_L &= \frac{1}{F}(\bar{h}l) = l_1 + \frac{1}{F}h_{(1)}l_1 \approx l_1 \\
e_L &= \frac{1}{F}(\epsilon^{ij}h_i l_j) = l_2 + \frac{1}{F}h_{(1)}l_2 \approx l_2 \\
u_L &= \frac{1}{F}(\bar{h}q) = q_1 + \frac{1}{F}h_{(1)}q_1 \approx q_1 \\
d_L &= \frac{1}{F}(\epsilon^{ij}h_i q_j) = q_2 + \frac{1}{F}h_{(1)}q_2 \approx q_2 \\
\phi &= \frac{1}{2F}(\bar{h}h) = h_{(1)} + \frac{F}{2} + \frac{1}{2F}h_{(1)}h_{(1)} \approx h_{(1)} + \frac{F}{2} \\
W^3_\mu &= \frac{2i}{gF^2}(\bar{h}D_\mu h) = \left(1 + \frac{h_{(1)}}{F}\right)^2 B^3_\mu + \frac{2i}{gF}\left(1 + \frac{h_{(1)}}{F}\right)\partial_\mu h_{(1)} \approx B^3_\mu \\
W^-_\mu &= \frac{\sqrt{2}i}{gF^2}(\epsilon^{ij}h_i D_\mu h_j) = \left(1 + \frac{h_{(1)}}{F}\right)^2 B^-_\mu \approx B^-_\mu, \\
W^+_\mu &= \left(\frac{\sqrt{2}i}{gF^2}(\epsilon^{ij}h_i D_\mu h_j)\right)^\dagger = \left(1 + \frac{h_{(1)}}{F}\right)^2 B^+_\mu \approx B^+_\mu,
\end{aligned} \tag{8}$$

where g is the coupling constant of the gauge group $SU(2)_L$ and D_μ is the corresponding covariant derivative. As can be seen from (8), the physical particles are those appearing in the standard model. This is a consequence of the complementarity between the Higgs phase and the confinement phase. We adopt the usual notation $B^\pm_\mu = (B^1_\mu \mp iB^2_\mu)/\sqrt{2}$. The terms which are suppressed by the large scale F are as irrelevant as the terms which are neglected in the Higgs phase when the Higgs field is expanded near its classical vacuum expectation value. If we match the expansion for the Higgs field

$\phi = h_{(1)} + \frac{F}{2}$ to the standard model, we see that $F = 2v = 492$ GeV where v is the vacuum expectation value. This parameter can be identified with a typical scale for the theory in the confinement phase. The physical scale is defined as $\Lambda = F/\sqrt{2}$, the $\sqrt{2}$ factor is included here because the physical parameter is not v but $v/\sqrt{2}$ as can be seen from the Lagrangian (1). We see in the expansion for W^3_μ that the suppressed irrelevant terms start at the order $2/F$. We thus interpret the typical scale for the confinement of W^3_μ as $\Lambda_W = \sqrt{2}F/4 = 173.9$ GeV.

3 A global SU(2) symmetry

In the absence of the $U(1)$ gauge group the theory has a global $SU(2)$ symmetry besides the local $SU(2)$ gauge symmetry. The scalar fields and their complex conjugates can be written in terms of two doublets arranged in the following matrix:

$$M = \begin{pmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{pmatrix}. \tag{9}$$

The potential of the scalar field $V(hh^*)$ depends solely on

$$\begin{aligned} h^*h &= h_1^*h_1 + h_2^*h_2 \\ &= (\mathrm{Re}\, h_1)^2 + (\mathrm{Im}\, h_1)^2 + (\mathrm{Re}\, h_2)^2 + (\mathrm{Im}\, h_2)^2 = -\mathrm{det}M. \end{aligned} \tag{10}$$

This sum is invariant under the group $SO(4)$, acting on the real vector $(\mathrm{Re}\, h_1, \mathrm{Im}\, h_1, \mathrm{Re}\, h_2, \mathrm{Im}\, h_2)$. This group is isomorphic to $SU(2) \otimes SU(2)$. One of these groups can be identified with the confining gauge group $SU(2)_L$, since $\mathrm{det}M$ remains invariant under $SU(2)_L$:

$$\det(UM) = \det(M), \ U \in SU(2)_L. \tag{11}$$

Now the second $SU(2)$ factor can be identified by considering the matrix $M^\top$

$$M^\top = \begin{pmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{pmatrix}. \tag{12}$$

The determinant of $M^\top$, which is equal to $\det(M)$, remains invariant under a $SU(2)$ transformation acting on the doublets (h_1, h_2^*) and $(h_2, -h_1^*)$.

These transformations commute with the $SU(2)_L$ transformations. They constitute the flavor group $SU(2)_F$, which is an exact symmetry as long as no other gauge group besides $SU(2)_L$ is present. With respect to $SU(2)_F$ the W-bosons form a triplet of states (W^+, W^-, W^3). The left-handed fermions form $SU(2)_F$ doublets. Both the triplet as well as the doublets are, of course, $SU(2)_L$ singlets. Once we fix the gauge in the $SU(2)_L$ space such that $h_2 = 0$ and $\mathrm{Im}\, h_1 = 0$, the two $SU(2)$ groups are linked together, and the $SU(2)_L$ doublets can be identified with the $SU(2)_F$ doublets. The global and unbroken $SU(2)$ symmetry dictates that the three W-bosons states, forming a $SU(2)_F$ triplet, have the same mass. Once the Yukawa-type interactions of the fields e_R, u_R and d_R with the corresponding left-handed bound systems are introduced, the flavor group $SU(2)_F$ is in general explicitly broken.

4 Electromagnetism and mixing

The next step is to include the electromagnetic interaction. The gauge group is $SU(2)_L \otimes U(1)_Y$, where Y stands for the hypercharge. The covariant derivative is given by

$$D_\mu = \partial_\mu - i\frac{g'}{2} Y \mathcal{A}_\mu - i\frac{g}{2}\tau^a B^a_\mu. \tag{13}$$

The assignment for Y is as follows:

$$\begin{aligned} Y \begin{pmatrix} l_1 \\ l_2 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} l_1 \\ l_2 \end{pmatrix} \\ Y \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} &= \begin{pmatrix} -\frac{1}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \\ Y \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}. \end{aligned}$$

The complete Lagrangian of the model in the confinement phase is given by:

$$\begin{aligned} \mathcal{L}_c = & -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \bar{l}_L i \not{D} l_L + \bar{q}_L i \not{D} q_L + \bar{e}_R i \not{D} e_R \\ & + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R + G_e \bar{e}_R (\tilde{h} l_L) + G_d \bar{d}_R (\tilde{h} q_L) \\ & + G_u \bar{u}_R (h q_L) + h.c. + \frac{1}{2} (D_\mu h)^\dagger (D^\mu h) - \frac{m^2}{2} h h^\dagger - \frac{\lambda}{4} (h h^\dagger)^2, \end{aligned} \tag{14}$$

where $m^2 > 0$ and

$$G^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g\epsilon^{abc} B^b_\mu B^c_\nu, \qquad (15)$$
$$f_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu.$$

The $U(1)$ gauge group is an unbroken gauge group, like $SU(2)_L$. The hypercharge of the h field is $+1$, and that of the h^* field is -1, i.e. the members of the flavor group $SU(2)_F$ have different charge assignments. Thus the group $SU(2)_F$ is dynamically broken, and a mass splitting between the charged and neutral vector bosons arises. The neutral massive vector boson which is proportional to $(\bar{h} D_\mu h)$ and which is not a gauge boson couples to the gauge boson $\mathcal{A}_\mu$, and as a result these bosons are not mass eigenstates, but mixed states. The strength of this mixing depends on the internal structure of the massive bosons.

We shall like to emphasize that the B^a_μ gauge bosons are as unphysical as the gluons are in QCD. The hyperphoton $\mathcal{A}_\mu$ is not the physical photon A_μ which is a mixture of $\mathcal{A}_\mu$ and of the bound state $(\bar{h} D_\mu h)$. The fundamental D-quarks do not have an electric charge but only a hypercharge. These hypercharges give a global hypercharge to the bound states, and one can see easily that a bound state like the electron has a global hypercharge and will thus couple to the physical photon, whereas a neutrino has a vanishing global hypercharge and thus will remain neutral with respect to the physical photon. So we deduce that QED is not a property of the microscopic world described by $\mathcal{L}_c$ but rather a property of the bound states constructed out of these fundamental fields. The theory in the confinement phase apparently makes no prediction concerning the strength of the coupling between the bound states and the electroweak bosons and the physical photon. This information can only be gained in the Higgs phase.

The mixing between the two states can be studied at the macroscopic scale, i.e. the theory of bound states, where one has

$$Z^0_\mu = W^3_\mu \cos\theta_W + \mathcal{A}_\mu \sin\theta_W$$
$$A_\mu = -W^3_\mu \sin\theta_W + \mathcal{A}_\mu \cos\theta_W. \qquad (16)$$

Here θ_W denotes the electroweak mixing angle, and A_μ denotes the photon field.

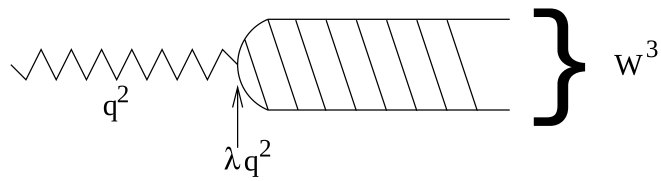

Figure 1: Hyperphoton transition into a W^3

5 The weak mixing angle

In this section, we want to calculate the typical scale for the confinement of the W^3_μ-boson. In section 2, we have matched the expansion for the Higgs field to the standard model. Using this point of view based on the effective theory concept, we obtained a scale of $\Lambda_W = 173.9$ GeV for this boson. Here we shall consider a effective Lagrangian to simulate the effect of the $SU(2)_L$ confinement.

This Lagrangian was originally considered in an attempt to describe the weak interactions without using a gauge theory [7]. This effective Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{eff} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{2}m^2_W W^a_\mu W^{a\mu} \\ & -\frac{1}{4}\lambda\left(F_{\mu\nu}W^{3\mu\nu} + W^3_{\mu\nu}F^{\mu\nu}\right)\end{aligned}$$

where we have

$$\begin{aligned}W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu, \\ F_{\mu\nu} &= \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu.\end{aligned} \tag{17}$$

The first term in the effective Lagrangian (17) describes the field of the hyperphoton, the second term three spin one bosons and the third term is a mass term which is identical for the three spin one bosons. In our case, the fourth term describes an effective mixing between W^3-boson and the hyperphoton.

The effective mixing angle of the Lagrangian given in equation (17) reads

$$\sin^2\theta = \frac{e}{g}\lambda. \tag{18}$$

Using the complementarity principle, we deduce that the mixing angle of the theory in the confinement phase has to be the weak mixing angle and therefore $\lambda = \sin\theta_W$.

The diagram in figure 1 enables us to relate the mixing angle to a parameter of the standard model in the confinement phase, the typical scale Λ_W for the confinement of the W^3-boson. For the annihilation of a W^3-boson into a hyperphoton we have the following relation

$$\langle 0|J_Y^\mu(0)|W^3\rangle = \frac{\epsilon^\mu}{\sqrt{2E_W}}\frac{m_W^2}{f_W} = \frac{\epsilon^\mu}{\sqrt{2E_W}} m_W F_W, \tag{19}$$

where J_Y^μ is the hyper-current, $F_W = m_W/f_W$ is the decay constant of the W^3-boson, and ϵ^μ is its polarization. The energy of the boson is E_W and the decay constant is defined as follows:

$$\lambda = \frac{e}{f_W}. \tag{20}$$

On the other hand, this matrix element can be expressed using the wave function of the W^3-boson which is a p-wave

$$\langle 0|J_Y^\mu(0)|W^3\rangle = \frac{\epsilon^\mu}{\sqrt{2E_W}}\sqrt{\frac{2}{m_W}}\partial_r\phi(0). \tag{21}$$

This leads to the following relation for the mixing angle

$$\sin^2\theta_W = \frac{8\pi\alpha}{m_W^5}\left(\partial_r\phi(0)\right)^2, \tag{22}$$

where $\alpha \approx 1/128$ at m_W is the fine structure constant.

We shall now consider two different models for the wave function:

a) Coulombic model.
We adopt the following ansatz for the radial wave function

$$\phi(r) = \frac{1}{\sqrt{3}}\left(\frac{1}{2r_B}\right)^{3/2}\frac{r}{r_B}\exp\left(-\frac{r}{2r_B}\right), \tag{23}$$

where r_B is the Bohr radius. Thus we obtain

$$r_B^{-1} = m_W\left(\frac{\pi\alpha}{3\sin^2\theta_W}\right)^{-1/5}. \tag{24}$$

If we define the typical scale for confinement as $\Lambda_W = r_B^{-1}$, we obtain $\Lambda_H = 157$ GeV.

b) Three-dimensional harmonic oscillator.
The radial part of the wave function is defined as follows:

$$\phi(r) = \sqrt{\frac{8}{3}} \frac{\beta^{3/2}}{\pi^{1/4}} \beta r \exp\left(-\frac{\beta^2 r^2}{2}\right), \tag{25}$$

where $\beta = \sqrt{m_W \omega}$, ω being the frequency of the oscillator. We identify the typical confinement scale Λ_W with the energy $E = \left(n + \frac{3}{2}\right)\omega$ corresponding to the quantum number of a p-wave i.e. $n = 1$, and we obtain

$$\omega = m_W \left(\frac{3 \sin^2 \theta_W}{64 \alpha \pi^{1/2}}\right)^{2/5} \tag{26}$$

and $\Lambda_W = \frac{5}{2}\omega = 182$ GeV.

Although we have performed a non-relativistic calculation, we see that the values we find for the typical composite scale are in good agreement with the naive guess we made based on the concept of an effective theory.

In order to estimate the value of $\sin^2 \theta_W$, we had to rely on the simple models, discussed above. However we should like to point out that $\sin^2 \theta_W$ is not a free parameter in our approach but fixed by the confinement dynamics. Thus the mixing angle can in principle be calculated taking e.g. the three dimensional harmonic oscillator:

$$\sin^2 \theta_W = \frac{256}{375} \sqrt{10\pi} \alpha \left(\frac{\Lambda_W}{m_W}\right)^{5/2}. \tag{27}$$

We can insert the value for Λ_W obtained from the effective theory point of view in equation (27) and we obtain $\sin^2 \theta_W = 0.2056$ which has to be compared to the experimental value $(\sin^2 \theta_W)_{exp} = 0.23124(24)$. Of course this is a naive non-relativistic and model dependent calculation. Such a calculation could also be done by lattice methods but it remains to see whether this can be carried out in the future.

6 Deviations from the standard model

In this section we discuss possible deviations from the standard model. A possible deviation from the standard model could be the ratio of the Higgs

boson mass to the W^3-boson mass. This ratio can take different values in the Higgs phase and in the confinement phase [8]. Thus the mass of the Higgs boson allows to test which phase describes the electroweak interactions.

Another aspect is the number of physical states. Obviously the particle spectrum of the model in the confinement is much richer than in the standard model since many new $SU(2)_L$ bound states can be constructed. Especially the d-waves, $D^1_{\mu\nu}$ and $D^2_{\mu\nu}$ of the bound state of two scalar D-quarks can be constructed. These bound states have the following expansions:

$$\begin{aligned} D^1_{\mu\nu} &= \frac{2i}{gF^2}(\bar{h}[D_\mu, D_\nu]h) = \left(1 + \frac{h_{(1)}}{F}\right)^2 G^3_{\mu\nu} \approx G^3_{\mu\nu} \\ D^2_{\mu\nu} &= \frac{2i}{gF^2}(\epsilon^{ij} h_i [D_\mu, D_\nu] h_j) = \left(1 + \frac{h_{(1)}}{F}\right)^2 \left(G^1_{\mu\nu} + iG^2_{\mu\nu}\right) \\ &\approx \left(G^1_{\mu\nu} + iG^2_{\mu\nu}\right), \end{aligned} \tag{28}$$

where $G^a_{\mu\nu}$ was defined in equation (15). The masses of such states could in principle be calculated using the lattice approach or at least related to the mass of the $W^\pm$-bosons. We also expect the appearance of bound states of the form $\epsilon^{ij} l_i l_j$, $\bar{l}l$, $\epsilon^{ij} q_i q_j$ and $\bar{q}q$ or the "mixtures" of the kind $\bar{l}q$ and $\epsilon^{ij} l_i q_j$. They have the following hypercharges

$$\begin{aligned} &Y(\epsilon^{ij} l_i l_j) = 2,\ Y(\bar{l}l) = 0,\ Y(\epsilon^{ij} q_i q_j) = -2/3,\ Y(\bar{q}q) = 0, \\ &Y(\epsilon^{ij} l_i q_j) = 2/3,\ Y(\bar{l}q) = -4/3. \end{aligned} \tag{29}$$

We cannot assign QED charges to these bound states since they are not present in the Higgs phase. We expect also the existence of radial excitations of known particles like a Z^* but basically their couplings to the rest of the particle spectrum and masses are unknown. But, we do not expect a sizeable coupling of these excited states to the fermions. They would be coupled primarily to the W-bosons, which consist of the same scalar D-quarks. A sizeable effect "beyond the standard model" could be observed in the reaction $W^+ + W^- \to Z^* \to W^+ + W^-$, a reaction which can be studied at the Tevatron and LHC colliders. The neutral d-wave state $D^1_{\mu\nu}$ discussed above could be found in the same reaction.

A major difference with composite models considered in the past is that we exclude four fermion interactions, which are setting strong constraints on composite models. The new particles, in our approach, should manifest

themselves as radiative corrections to known processes. We should nevertheless keep in mind that not even the Z-boson contribution to the anomalous magnetic moment of the muon can be identified [9].

Let us return to the problem of quantum electrodynamics. As long as the $SU(2)_L$ confinement takes place we cannot introduce the charge operator

$$Q = 1/2(\tau^3 - Y) \tag{30}$$

since it is not a singlet under $SU(2)_L$. But at very small distances, when the confinement scale is irrelevant, this operator becomes physical and in principle observable. We can rewrite the covariant derivative (13) using the mass eigenstate basis. Since the gauge symmetry is not broken we are not forced to do so, but we have the freedom to use this basis. We have

$$\begin{aligned} D_\mu = \partial_\mu - i\frac{gg'}{2\sqrt{g^2+g'^2}}A_\mu(\tau^3 - Y) - i\frac{1}{2\sqrt{2}}g(\tau^+ B^+_\mu + \tau^- B^-_\mu) \\ -i\frac{1}{2\sqrt{g^2+g'^2}}Z_\mu\left(g^2\tau^3 + g'^2 Y\right), \end{aligned} \tag{31}$$

with

$$\tau^\pm = 1/2(\tau^1 \pm i\tau^2). \tag{32}$$

Thus the electric charges of the leptonic D-quarks, hadronic D-quarks and scalar D-quarks are

$$\begin{aligned} Q\begin{pmatrix} l_1 \\ l_2 \end{pmatrix} &= \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} l_1 \\ l_2 \end{pmatrix} \\ Q\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} &= \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \\ Q\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} &= \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}. \end{aligned} \tag{33}$$

If one would be able to test the charge of an electron D-quark at very small distances, we would measure the right QED charge. We can then deduce that at low energies it is basically not possible to distinguish the two phases as the known particles are expected to behave in the same fashion as in the standard model, and the new excited bound states cannot be produced

because they are too heavy. At intermediary energies one can expect the new bound states to contribute through quantum processes but at very small distances, the bound states will disappear, the D-quarks will become free, and they will have the usual charges as already discussed. The gauge bosons of the $SU(2)_L$ gauge group will also be liberated and take over the role of the electroweak bosons which were bound states. Especially the gauge boson B^3_μ will mix with the hyperphoton, and it will become impossible to distinguish between the two phases.

A frequent problem with models avoiding the Higgs mechanism are flavor changing neutral currents. Our model can be extended to three families by introducing the following doublets in the model

$$l_i^{(2)} = \begin{pmatrix} l_1^{(2)} \\ l_2^{(2)} \end{pmatrix},\ l_i^{(3)} = \begin{pmatrix} l_1^{(3)} \\ l_2^{(3)} \end{pmatrix},\ q_i^{(2)} = \begin{pmatrix} q_1^{(2)} \\ q_2^{(2)} \end{pmatrix} \text{and } q_i^{(3)} = \begin{pmatrix} q_1^{(3)} \\ q_2^{(3)} \end{pmatrix}. \tag{34}$$

We can then identify the leptons and quarks of the second family in the following way

$$\begin{aligned}
\nu_{\mu\, L} &= \frac{1}{F}(\bar{h} l^{(2)}) = l_1^{(2)} + \frac{1}{F} h_{(1)} l_1^{(2)} \approx l_1^{(2)} \\
\mu_L &= \frac{1}{F}(\epsilon^{ij} h_i l_j^{(2)}) = l_2^{(2)} + \frac{1}{F} h_{(1)} l_2^{(2)} \approx l_2^{(2)} \\
c_L &= \frac{1}{F}(\bar{h} q^{(2)}) = q_1^{(2)} + \frac{1}{F} h_{(1)} q_1^{(2)} \approx q_1^{(2)} \\
s_L &= \frac{1}{F}(\epsilon^{ij} h_i q_j^{(2)}) = q_2^{(2)} + \frac{1}{F} h_{(1)} q_2^{(2)} \approx q_2^{(2)}.
\end{aligned} \tag{35}$$

The fermions of the third family are

$$\begin{aligned}
\nu_{\tau\, L} &= \frac{1}{F}(\bar{h} l^{(3)}) = l_1^{(3)} + \frac{1}{F} h_{(1)} l_1^{(3)} \approx l_1^{(3)} \\
\tau_L &= \frac{\sqrt{2}}{F}(\epsilon^{ij} h_i l_j^{(3)}) = l_2^{(3)} + \frac{1}{F} h_{(1)} l_2^{(3)} \approx l_2^{(3)} \\
t_L &= \frac{1}{F}(\bar{h} q^{(3)}) = q_1^{(3)} + \frac{1}{F} h_{(1)} q_1^{(3)} \approx q_1^{(3)} \\
b_L &= \frac{1}{F}(\epsilon^{ij} h_i q_j^{(3)}) = q_2^{(3)} + \frac{1}{F} h_{(1)} q_2^{(3)} \approx q_2^{(3)}.
\end{aligned} \tag{36}$$

The electroweak and Higgs bosons are defined in the same way as previously. The universality of the weak interactions is respected as the three families couple with the same strength to the electroweak bosons. Due to the complementarity between both phases of the model we do not expect new physical effects as for example flavor changing neutral currents. So this model reproduces all the phenomenological success of the standard model.

We are aware that the new effects we have discussed in this section could be forbidden or driven up to very high energies by some yet unknown selection rules. However this looks unlikely since it does not appear to be very natural. Another possibility is that these non-perturbative effects are also present, but hidden in the standard model Lagrangian in which case the complementarity would not break down.

7 Conclusions

We have considered a model based on the gauge group $SU(2)_L \otimes U(1)_Y$. This model represents a viable alternative to the electroweak standard model. The basic assumption is that physical particles are singlets under $SU(2)_L$. The model allows a dynamical interpretation of the weak mixing angle which is related to a typical scale for the confinement of the W^3-boson. This is the main achievement of the model in the confinement phase. This scale can be calculated by means of simple wave functions. The result of this naive calculation agrees with the result expected from the effective theory point of view. In our approach, the weak mixing angle and therefore also the ratio m_Z/m_W are calculable.

Some deviations from the standard model could be observed but any prediction is highly dependent on the details of our confining theory, whose non-perturbative effects are yet to be investigated. Lattice theory should be able to shed some light onto these questions.

If our interpretation is correct, all gauge theories describing the elementary interactions are based on exact gauge symmetries. This would restore the asymmetry between QCD and the standard model. Unless there are some unknown selection rules forbidding their presence, we expect a rich new spectrum of bound states which could appear at the scale of a few hundred GeV. If the confinement phase describes the electroweak interactions, the desert in the TeV region could be a blooming desert full of interesting new phenomena.

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